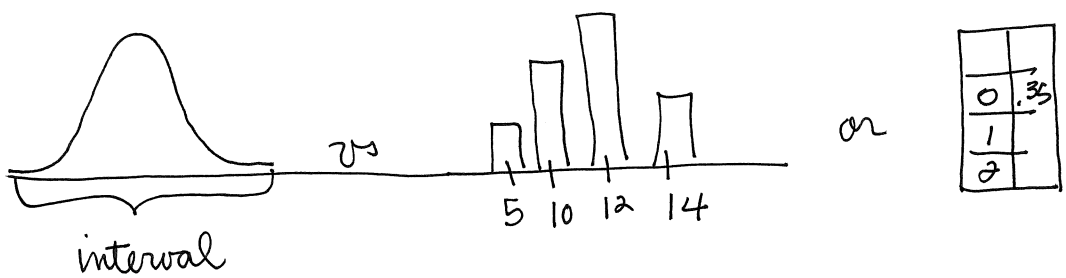
7.5 The Normal Curve and just a bit of 7.6

The standard normal curve

the bell curve
symmetric, mound shaped,
continuous

Let's discuss continuous versus discrete again

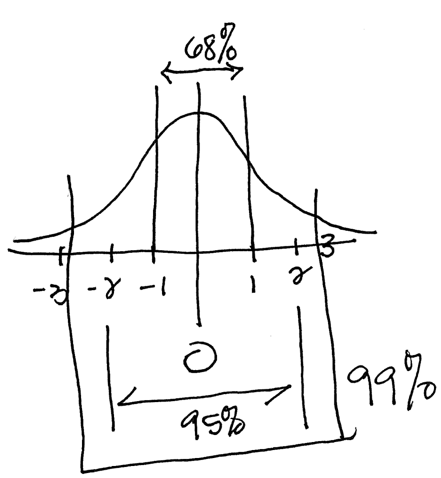


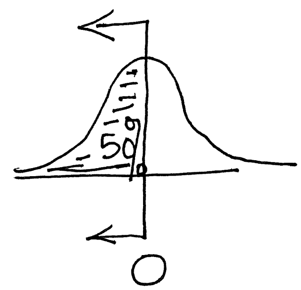
For the standard normal curve the mean is zero and the standard deviation is 1.

$N(0,1)$

It is symmetric about **z** = 0…not x? why?
Probabilities correspond to area under the curve. and we use zscore to find them.

Let's review the Empirical Rule (p. 71) right now with a picture:





Now let's look at the standard normal probability table. $N(0,1)$ back of the book

Given a z-score of 1.28, what is the probability that a measurement is at or below this value?

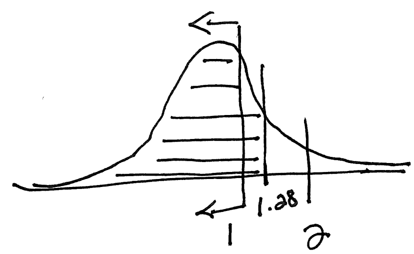
page 210 rows: 1.2 across to under .08 Area is .8997 at or below this z score.

.00 .01 ... .08

1.0
1.1
→ 1.2 → $\boxed{.8997}$
1.3

$P(Z \leq 1.28) \approx 90\%$

at a value or <u>less</u> on the chart

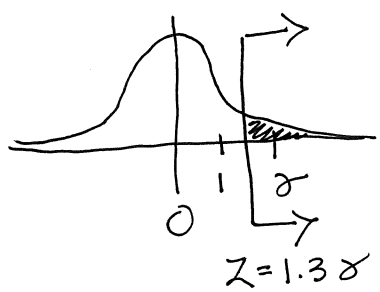


Now for using the chart with "greater than or equal to"…a version of the complement rule! P(Z >1.32) = 1 - .9066 = .0934

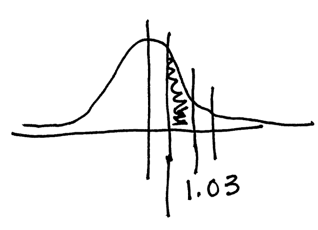
The complements rule

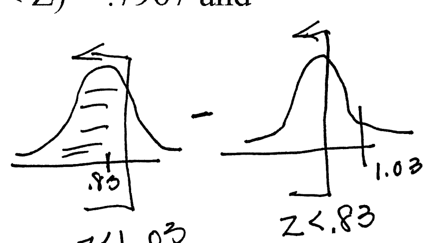
Area under the upper tail is .0934

$P(Z \geq 1.32) \approx 9\%$



Or between two measurements! P(.83< Z < 1.03) P(.83 < Z) = .7967 and P(Z < 1.03) = .8485





.8485 − .7967

P between the z's is .8485 − .7967 = .0518 ≅ 5%

Using the table in reverse: from a probability to a z-score:

Page 212 Suppose the P is 61%...look in the table…find .6064 and .6103 look up and across:
.26 and.27 for z scores. Since between can extrapolate to .267 or .268 (closer to .27)

.06 · .07

61st percentile

0.20 ← .6064 . .6013

inbetween

$Z \approx .267$

In reality, MOST normal curves are NOT standard! How do we rescale to make use of our standard normal chart? With z-scores! All normal curves are proportional and we use the z-score calculation to make them "fit" the table.
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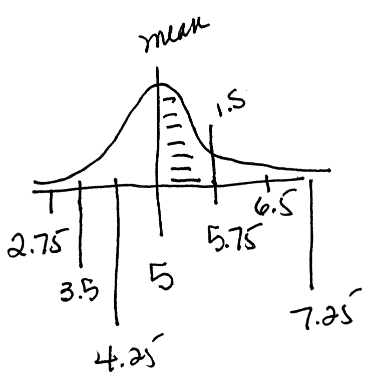
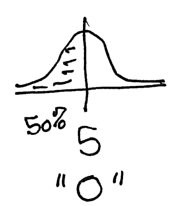
Let's look at a normal curve with mean 5 and sd .75. What's the area between 1 sd above ~~and below~~ the mean? Empirical Rule. What are those measurements for THIS curve. How will we use z-score to discover this on the chart for the standard normal distribution?

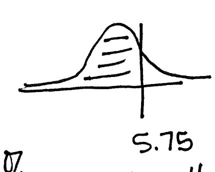


$Z_{5.75} = 1.0$

2 ways

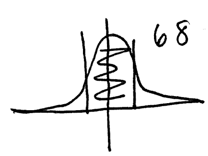


take away from



$.84 - .5 \cong 34\%$

or



68% ÷ 2 34%

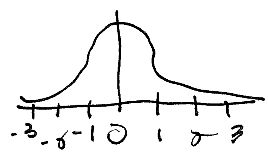
From another source – TI83 instructions for

Areas between two bounds:

2nd VARS   [2: normal cdf(left z score, right z score)]

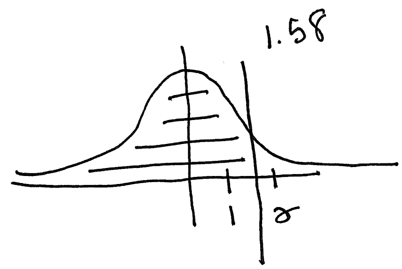
Normal Distributions: together!

The Precision Scientific Instrument Company manufactures thermometers. To check the accuracy, they test the thermometers in freezing water and make sure it registers 0 degrees F. Of course some are high and some are low. Assume there is a standard deviation of 1 degree F. Find the area and show it on a standard normal curve!



What is the probability that the reading is less than 1.58°?

You should get 94.29%



.08

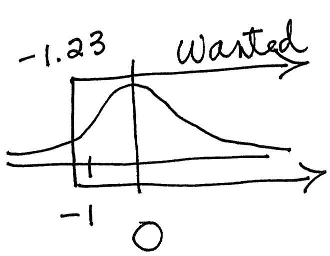
↓

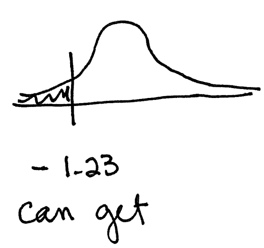
1.5 → .9429

What is the probability that the reading is above −1.23°?

You should get .8907

complements rule





$1 - P(Z \leq -1.23) =$

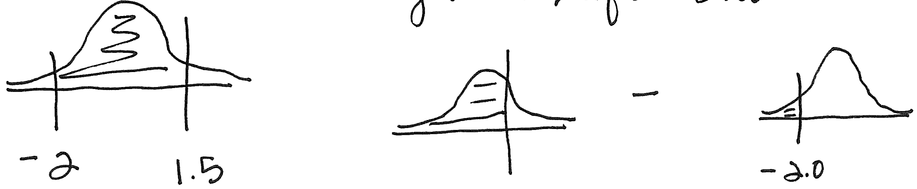
89 ish %

What is the probability that the reading is between −2° and 1.5°?

You should get 91.04%

want

get 1.5 & left subtract 1.2 & left



0.0 ↓ 1.5→

0.0 ↓ −2.0→

Working backwards in the chart: ↓ between 2 values

Find the temperature associated with the 95th percentile. $z = 1.645$

How does this work?

↑ ← .9500 or close

Find the temperatures separating the bottom 2.5% and the top 2.5%

These are called tolerances. (−1.96 and 1.96 for z's).

How does this work?

find .06 ↑ 1.9 ← .025

z score is temp in this problem

by symmetry same below

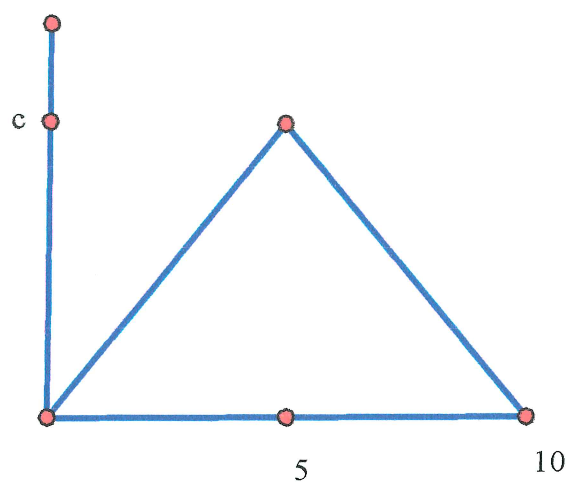
Fill in the blanks:

About __68%__ % of the area is within 1 standard deviation of the mean   above and below

About __95%__ % of the area is within 2 standard deviations of the mean

About __99%__ % of the area is within 3 standard deviations of the mean

Enrichment: The Triangle Distribution. Manufacturing fill problems



Area under the curve is 1 (100%)

$A_T = \frac{1}{2}bh$

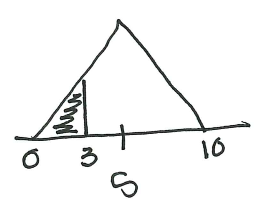
$1 = \frac{1}{2}(10)h$

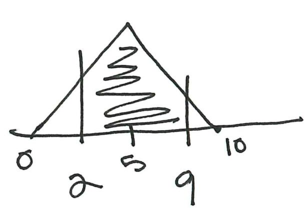
$\frac{1}{5} = h \qquad c = \frac{1}{5}$

Here is a probability distribution. Find the value of c.

Show the probability that x is between 0 and 3.

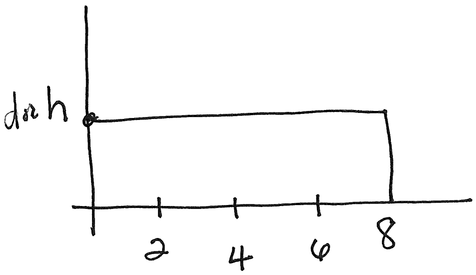
Show the probability that x is between 2 and 9.





UNIFORM Distribution

Sketch one and find d for a horizontal axis from 0 to 8



$A_r = bh$

$1 = 8 \cdot h$

$h = 1/8$

note $P(Z \leq 2) = 25\%$

$P(Z \geq 6) = 25\%$

ND problem 1 — finish for homework A

Air Force ejection seats are designed for people weighing between 140 lb and 211 lb. Women's weights are normally distributed with a mean of 143 lb and a standard deviation of 29 lb. What percentage of women have weights in those limits?

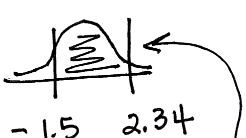
$P(140 \leq \overset{\text{pilot}}{\text{Weight}} \leq 211) = 1$

$P(140 \leq \text{women's weight} \leq 211) = P\left(\frac{140-143}{29} \leq Z \leq \frac{211-143}{29}\right)$

1.5 2.34

$P(-1.5 \leq Z \leq 2.34)$

you start here



we want this

Homework problem 2

The lengths of pregnancies are normally distributed with a mean of 268 days and a standard deviation of 15 days. $N(268, 15)$

A woman wrote to Dear Abby claiming that she gave birth 290 days after a brief visit with her husband who was fleet Navy and ship bound else. Is this credible?

X – getting pregnant, # days to having the baby

Again, we'll start together

you start here

$$P(X < 290) = P(X \leq 290) = P\left(Z \leq \frac{290-268}{15}\right)$$

Premature is being born in the 4^{th} percentile of length of pregnancy…what length of time is this?

Hint: use the z score formula to back solve for the days because you'll know the z score but not the X from the table

want this

$\leftarrow .04$

$$Z = \frac{x - \mu}{\sigma}$$

Finishing up the poppers

Chapter 7 Popper Question 5

The z score associated with the 55 percentile is approximately:

A. 0.15 B. 0.21

Chapter 7 Popper Question 6

To work a probability problem with a Normal Distribution that is NOT the Standard Distribution, you set the problem up with X and then convert it to Z using the z score formula and the non-standard mean and standard deviation.

A. True B. False

Chapter 7 Popper Question 7

To find the $P(Z > .91)$, use the Complements Rule: $1 - P(Z < .91)$

A. True B. False

Chapter 7 Popper Question 8

All normal distributions are proportional and the probability that X is between 1 and – 1 is 68% for them all.

A. True B. False

Chapter 7 Popper Question 9

Given N(7, 2), the Probability that X is between 3 and 11 is 95%.

A. True B. False

Now for a brief summing up:

We looked first at Discrete Distributions – both individual ones that pertained to just one problem and the Binomial Distribution. In all cases, there was a mean and a standard deviation, and the distribution probabilities added up to one.

We used tree diagrams and area charts and frequency data to find the probabilities and then summarized them in tables with a finite number of rows.

We can have negative values for X even though we didn't work with any of these. These are in the first column of the distribution. The second column is made up of probabilities and these are all positive numbers between and including 0 and 1.

The Binomial Distribution is a general case distribution. You need

Repeated identical trials, n of them
Independent trials
Exactly two outcomes (P and Q)
$p + q = 1$
x is the number of successes and n – x is the number of failures

The formulas for an individual discrete distribution m and sd are DIFFERENT than those for the Binomial Distribution.

We looked at 3 Continuous Distributions: Normal, Triangle, and Uniform. These are all in Quadrants 1 and 2 and over an interval of the horizontal axis. Finding probabilities is a matter of finding the area between, to the left, or the right of a given value. Less than or equal to has the SAME probability as a straight less than because adding in a line does NOT add area. This is not usually true with Discrete Distributions. Continuous Distributions have the area under the curve = 1 and they each have a mean and a standard deviation too. N(0,1) is the Standard Normal Distribution and is generalized to cover all scenarios that are normally distributed.

Summary of Chapter 7: A 9 question popper, no essays, 2 homework problems to complete in 7 C plus from the book: 2, 6, 11, 18 (only a and f).